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A NOTE ON SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS USING POWER SERIES

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ABSTRACT

The method of power series solution is a traditional but strong method for Ordinary Differential Equations and Partial Differential Equations. However, despite their usefulness the application of this method has been limited to this particular kind of equations. We propose to use the method of Power Series to solve Non Linear Partial Differential Equations. We apply the method in several typical non linear partial differential equations in order to demonstrate the power of the method. This method ensures the theoretical exactness of the approximate solution and comparisons of the approximate solution with the exact one are determined.

KEY WORDS: *Ordinary Differential Equations, Non Linear Partial Differential Equations, Power Series.*

INTRODUCTION:

In present situation, the solution of non-linear partial differential equations is considered as a fundamental tool in the research of multidisciplinary areas, because both their implication in the public health problems and social impact in to solve real life problems. In fact, is mandatory to involve mathematical methods in the traditional research methodology of science areas like Biology, Cell Biology, Physiology, Physics, Chemistry, Chemical Physics and different branch of engineering etc. which helped by the technological advance in the computation, to incorporate a new age of knowledge in order to tackle real life problems.

Power Series Solution method has been limited to solve Linear Differential equations, both Ordinary [1, 2], and Partial Differential Equation [3, 4]. Linear Partial Differential Equation has traditionally been solved using the variable separation method because it permits to obtain a coupled system of Ordinary Differential Equation easier to solve with the Power Series

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Solution method. Examples of these are the Legendre polynomials and the spherical harmonics used in the Laplace´s Equations in spherical coordinates or the Bessel´s equations in cylindrical coordinates [3-4]. It is known that in Non Linear Partial Differential Equation is, as we know, because isn´t possible to apply the separation of variables method.

The methodology to solve the in Non Linear Partial Differential Equation is to obtain a solution by using the approximated analytical method, i. e., non numerical or semi analytic form, in an indirect or direct way. In the direct way, there are methods like Inverse Scattering Transform [5] or the Lax Operator Formalism [6]. In the direct way it can be used for example the Power Series Solution in an asymptotic approximation the Hirota method involving a bilinear operator technique [7], the Adomian Decomposition Method [8], and the Homotopy Analysis method [9, 10]. This last method involves a series expansion with a non small parameter perturbation approximation to adjust the convergence. This method is different of the classical perturbation theory. Techniques, even more direct, to approximate to a solution in Non Linear Partial Differential Equation are the Taylor Polynomial Approximation method [11, 12], and the Power Series Solution Method. In both techniques a semi analytic solution is obtained implementing the Power Series Solution Method. However, the Power Series Solution Method has been little used to solve non-linear Ordinary Differential Equation [13-16], or Power Series Solution Method [17-19].

Here we have tried to verify the power series solution by considering some Non Linear Partial Differential Equation which has exact solution.

METHODOLOGY: POWER SERIES SOLUTION

In mathematics the power series is an infinite series that can be written as a polynomial with an infinite number of terms. In general the power series [20] can be written as,

$$\sum_{n=0}^{n=\infty} a_n x^n = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \dots\dots\dots\dots\dots$$

where is a x variable and a_n are constants which are called co-efficient of the series. The power series will converge [21] for all the values of x in the series within certain interval, particularly, whenever the absolute value of x is less than some positive number r, known as radius of convergence.

In this paper we concentrate on the power series of two variables which can be written in the form of Taylor series. These series also have a centre where they always converge to the function value. If the centre be (x_0, y_0), then the series of two variables in the basic polynomial form is given by

$\sum a_{ij} (\bar{x})^i (\bar{y})^j$ where $\bar{x} = x - x_0, \bar{y} = y - y_0$

where the indices i and j run (independently) from 0 to 1. For instance , the terms up to order 2 can be written as,

$a_{00} + a_{10}(x - x_0) + a_{01}(y - y_0) + a_{20}(x - x_0)^2 + a_{11}(x - x_0)(y - y_0) + a_{02}(y - y_0)^2$

In Calculus one learns to understand the behaviour of all such quadratic polynomials in two variables. In order to write down a Taylor series completely, we need to explain how to get the coefficients a_{ij} in terms of the function. As before, the zero order or constant term is just $a_{00} = f(x_0, y_0)$ and the other terms involve derivatives of the function f evaluated at (x_0, y_0) and factorials.

Let us explain by considering an example, $f(x_0, y_0) = \tan^{-1} x$ at the centre point $(x_0, y_0) = (1,1)$.

To get a_{10} , treat y as a constant, and differentiate with respect to x . Substitute the centre (x_0, y_0) into the resulting expression. The expression that is partial derivative w.r.t. x obtained by differentiating is denoted by f_x at x and may also simply be called the "first x-partial," or the "first homogeneous x -partial".

Next by evaluating this expression at $(x_0, y_0) = (1,1)$. we obtain the coefficient of the first order linear term in x is $a_{10} = -\frac{1}{2}$. In the same manner we can find a_{ij} for different values of i and j .

Here our intension is to check the solution obtained by power series which is same as its exact solution. So now by applying this power series method, we try to verify the series solution of the following examples which have already exact solutions.

EXAMPLE:

Consider the partial differential equation $px + qy = pq$ .... (1)

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Where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ are the partial derivatives.

The exact solution of this equation by Charpits method (22) is given by,

$$2cz = (cx + y)^2 + 2a \quad ....(2)$$

Where a and c are arbitrary constant. This equation can also be written as

$$z = \frac{a}{c} + xy + \frac{c}{2}x^2 + \frac{1}{2c}y^2 + ... \quad ....(3)$$

Equation (3) is in the series form. So we try to obtain the series solution using power series method which matches the equation (3).

Now let us assume the solution of (1) as a power series in x ,and y as below,

$$z(x, y) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} a_{i\,j} x^i y^j \quad .... (4)$$

By differentiating both sides of equation (4) with respect to x and y we get the series expansion of p and q as follows,

$$p = \frac{\partial z}{\partial x} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} a_{(i+1)j}(i+1)x^i y^j \quad ....(5)$$

$$q = \frac{\partial z}{\partial y} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} a_{i(j+1)}(j+1)x^i y^j \quad ....(6)$$

$$pq = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\sum_{p=0}^{i}\sum_{q=0}^{j}(p+1)(j-q+1)a_{(p+1)q}a_{(i-p)(j-q+1)}x^i y^j\right] ....(7)$$

Substituting (5),(6)(7) into equation (1), we obtain the following recurrence relation,

$$a_{ij} = \frac{1}{i+j}\left[\sum_{p=0}^{i}\sum_{q=0}^{j}(p+1)(j-q+1)a_{(p+1)q}a_{(i-p)(j-q+1)}\right] .... (8)$$

Where

$$a_{i0} = \frac{a}{c} \text{ if } i = 0 \quad \text{and} \quad a_{i0} = \frac{c}{2} \text{ if } i = 2,\ a_{i0} = 0 \text{ otherwise} ....(9)$$

By applying the recurrence relation (8)for several values of i and j the polynomial approximate series solution obtained is as follows,

$$z = \frac{a}{c} + xy + \frac{c}{2}x^2 + \frac{1}{2c}y^2 + ... \quad ....(10)$$

Comparing equation (10)with equation (3), the power series solution exactly matches with exact solution obtained by Charpits method.

Result and Conclusion:

In this paper we verify that, it is possible to solve non-linear partial differential equation with the power series method. Here the solution of equation (1) obtained by power series method exactly matches with the exact solution (3). This verification forces us to conclude that the method works for almost all problems.

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